MAP 6932/6905 Seminar in Applied Mathematics

Introduction to Convex Optimization and Applications in Image Analysis (part II)

References:

Introductory Lectures on Convex Optimization, by Y. Nesterov, Kluwer, Boston, 2004

R. T. Rockafellar, Convex Analysis, Princeton University Press, Princeton, NJ, 1970.

Mathematical Problems in Image Processing – PDE and the Calculus of Variations by Gilles Aubert and Pierre Kornprobst;

The Handbook of Mathematical Models in Computer Vision by Nikos Paragios, Yunmei Chen, and Olivier Faugera

Related papers:

1. Beck and M. Teboulle, A fast iterative shrinkage-thresholding algorithm for linear inverse problems, SIAM J. Imaging Sci., 2 (2009), pp. 183–202.
2. Beck and M. Teboulle, Gradient-based algorithms with applications to signal recovery problems. In: Convex Optimization in Signal Processing and Communications, Edited by Y. Eldar and D. Palomar, Cambridge University Press, (2010).
3. Y. Nesterov. A method for solving the convex programming problem with convergence rate O(1/k2). Dokl. Akad. Nauk SSSR, 269(3):543–547, 1983.
4. Y. Nesterov, Smooth minimization of non-smooth functions, Math. Program., 103 (2005), pp. 127–152.
5. Y. Nesterov, Gradient methods for minimizing composite objective function, http://www.ecore.beDPs/dp1191313936.pdf (2007).
6. P. Tseng, On Accelerated Proximal Gradient Methods for Convex-Concave Optimization, Technical report, Department of Mathematics, University of Washington, Seattle, WA; available online from http://www.math.washington.edu/~tseng/papers.html.

Meeting Time and Rooms*:*

MWF 5 at LIT 207

Office Hours: MWF 4 or by appointment

Objective and Outline of the Course:

This course is a continuation of the introduction to the the gradient based methods and their convergence analysis for solving nonlinear models arising from data/image analysis given in the first part of the course. In this course we will study several classical numerical methods for minimizing certain classes of convex smooth or non-smooth functionals with optimal rate of convergence. Those methods include Nesterov's multi-step fast gradient method for smooth convex optimizations, Nesterov's smoothing techniques for non-smooth optimizations, fast proximal gradient method, accelerated primal dual method, accelerated alternating direction method of multipliers, accelerated bundle level method, and their variants. We will also analyze the rate of convergence of those schemes. Moreover, we will study certain line search techniques for those optimal schemes to improve their performances in practice. Students are expected to gain knowledge on mathematical theories, methods, and practical experience in solving real world problems.

Arrangement of the course:

Unit 1: Sub-gradients, Convex Conjugates:

1. Sub-gradients: definition, properties, and computation rules;

2. Optimality conditions;

3. Convex conjugate: definition, properties, and computation rules.

(Tentative weekly schedule: week 1-2)

Unit 2: Nesterov's Optimal Gradient Methods

1. Optimal gradient methods for minimizing smooth convex functions;

2. Smoothing techniques for minimizing non-smooth convex functions.

3. Min-max problems, optimal scheme, duality map, and rate of convergence.

(Tentative weekly schedule: week 3-5)

Unit 3: Accelerated Bundle Level Methods

1. Bundle level method: scheme, rate of convergence;

2. Accelerated bundle level method: scheme, rate of convergence;

3. Fast accelerated bundle level method: scheme, comparison with accelerated bundle level method in performances.

4. Applications to solve smooth convex minimization and min-max problems

2. Accelerated bundle level method: scheme, rate of convergence;

3. Fast accelerated bundle level method: scheme, comparison with accelerated bundle level method in performances.

4. Applications to solve smooth convex minimization and min-max problems.

(Tentative weekly schedule: week 6-8)

Unit 4: Proximal Gradient Method:

1. Proximal gradient method for minimizing a composite function of a smooth function and non-smooth function;

2. Key inequalities and rate of convergence;

3. Fast proximal gradient method (fast iterative shrinkage thresholding algorithm – FISTA) versus proximal gradient method (iterative shrinkage thresholding algorithm – ISTA): scheme, and rate of convergence.

(Tentative weekly schedule: week 9-10)

Unit 5: Accelerated Primal Dual Method (APD) , Accelerated Alternating Direction Method of Multiplies (AADMM), and Their Applications to Image Analysis

1. Primal Dual method and APD schemes, convergence analysis, and applications to total variation based image reconstruction;

2. Alternating Direction Method of Multiplies (ADMM) and accelerated ADMM: schemes, convergence analysis, and applications to image analysis.

(Tentative weekly schedule: week 11-13)

Unit 6: Some Backtracking Techniques for APD and AADMM, and Their Applications

1.Bregman operator splitting (BOS) with constant and BOS with variable step sizes (BOSVS): schemes, convergence analysis;

2. Accelerated BOSVS: schemes, convergence analysis, and applications to total variation based image reconstruction;

3. Comparisons of the methods discussed in this semester.

(Tentative weekly schedule: week 14-16)

Grading:

Students will be required to present one to two papers and the projects related to the course content. These projects may be related to problems of particular interest to the individual student. Grades will be assigned on the basis of these projects. Current UF grading policies can be found from the following link http://www.registrar.ufl.edu/catalog/policies/regulationgrades.html

Teaching Evaluation:Students are expected to provide feedback on the quality of instruction in this course based on 10 criteria. These evaluations are conducted online at https://evaluations.ufl.edu .

Academic Honesty: The course will be conducted in accordance with the University honor code and academic honesty policy, which can be found in the student guide

Accommodation for Student with Disabilities: Students requesting classroom accommodation must first register with the Dean of Students Office. The Dean of Students Office will provide documentation to the student who must then provide this documentation to the Instructor when requesting accommodation.